

Weak QMV algebras and some ring-like structures

弱QMV代数以及类环结构

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1. Background & Motivation

Quantum logic was introduced in the 1930s as the logic of quantum systems. Algebras corresponding to all positive operators are effect algebras and quantum many-valued (QMV) algebras, which are called unsharp quantum logics because of the lack of the noncontradiction principle. A study of ring-like structures was initiated to find a most general framework for developing axiomatic quantum mechanics. It was shown that coupled semirings are equivalent with MV algebras, then what are the ring-like quantum structures corresponding to QMV algebras?

2. Weak QMV algebras (wQMV algebras)

The weak QMV algebra is defined as a QMV algebra, except that the axiom

$$a \oplus [(a^* \sqcap b) \sqcap (c \sqcap a^*)] = (a \oplus b) \sqcap (a \oplus c)$$

is replaced by

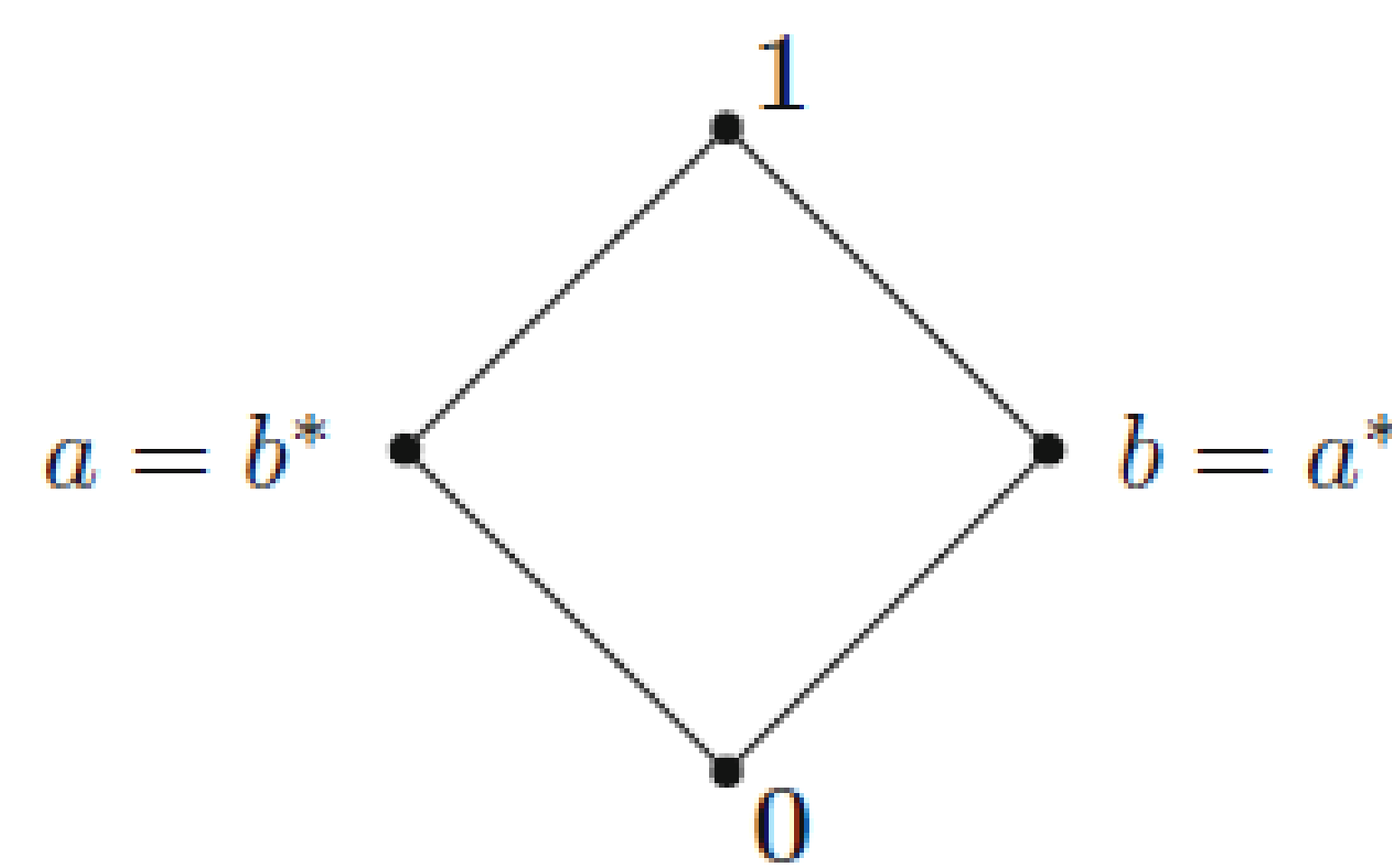
$$a \oplus [(b \sqcap a^*) \sqcap (c \sqcap a^*)] = (a \oplus b) \sqcap (a \oplus c)$$

which weakens the distributive law of QMV algebras. We also proved that a wQMV algebra is a QMV algebra iff $(a \oplus b) \sqcap (a \oplus c) = a \oplus (b \sqcap c)$ for all elements. Most properties, such as monotony and cancelation law, are reversed by wQMV algebras.

3. Automatic generation of finite algebras

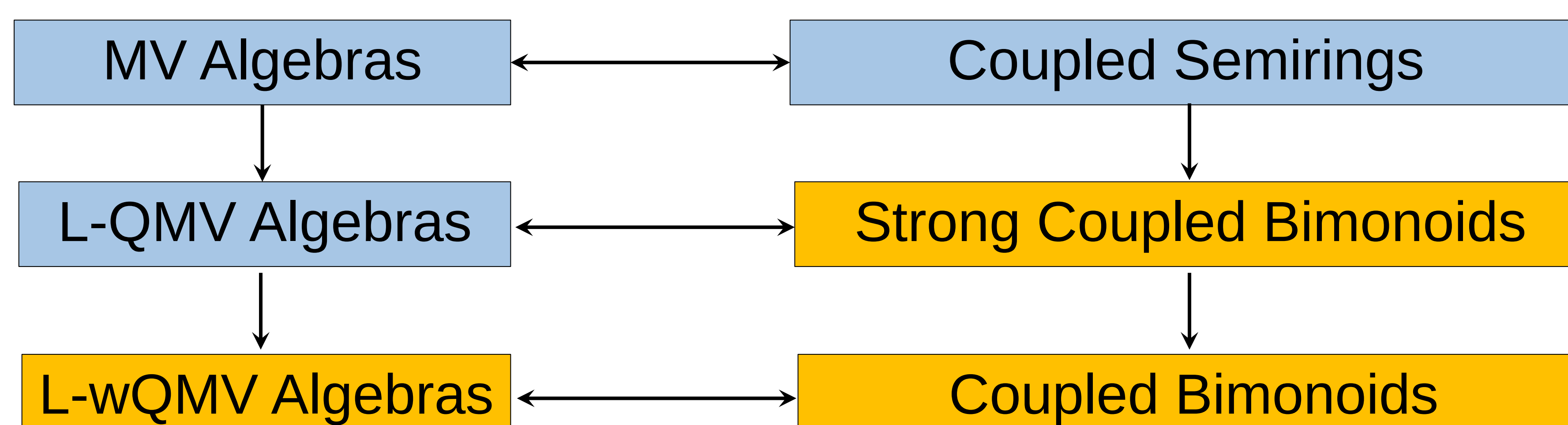
QMV algebras are properly contained in the wQMV algebras. We can use a tool called SEM to generate a smallest wQMV algebra but not a QMV algebra, as

$\oplus$	0	1	a	b
0	0	1	a	b
1	1	1	1	1
a	a	1	1	1
b	b	1	1	b



4. Coupled bimonoids

Since wQMV algebras are defined by relaxed the distributive law, we also relax the distributive law in the coupled semirings, which leads to a complete diagram:



We proposed l-wQMV algebras, coupled bimonoids and strong coupled bimonoids. We also proved that the strong coupled bimonoids are the answer algebras. As an application of sharp quantum logical structures, Ying, Qiu, etc. set up the computation theory based on sharp quantum logics, Lu and Shang set up the computation based on unsharp quantum logics. The lattice-ordered wQMV algebras we proposed are weaker unsharp quantum structure based on which computation theory could be established.